

Inference at \* 1 2 1  
of proof for Lemma fast-fib:

1.  $n : \mathbb{Z}$
  2.  $0 < n$
  3.  $\forall a, b: \mathbb{N}.$   
 $\{m: \mathbb{N} \mid$   
 $\forall k: \mathbb{N}.$   
 $(a = \text{fib}(k))$   
 $\Rightarrow ((k \leq 0) \Rightarrow (b = 0))$   
 $\Rightarrow ((0 < k) \Rightarrow (b = \text{fib}(k - 1)))$   
 $\Rightarrow (m = \text{fib}((n - 1) + k))\}$
  4.  $a : \mathbb{N}$
  5.  $b : \mathbb{N}$
  6.  $\forall b_1: \mathbb{N}.$   
 $\{m: \mathbb{N} \mid$   
 $\forall k: \mathbb{N}.$   
 $(a + b = \text{fib}(k))$   
 $\Rightarrow ((k \leq 0) \Rightarrow (b_1 = 0))$   
 $\Rightarrow ((0 < k) \Rightarrow (b_1 = \text{fib}(k - 1)))$   
 $\Rightarrow (m = \text{fib}((n - 1) + k))\}$
- $\vdash \{m: \mathbb{N} \mid$   
 $\forall k: \mathbb{N}.$   
 $(a = \text{fib}(k))$   
 $\Rightarrow ((k \leq 0) \Rightarrow (b = 0))$   
 $\Rightarrow ((0 < k) \Rightarrow (b = \text{fib}(k - 1)))$   
 $\Rightarrow (m = \text{fib}(n + k))\}$   
by InstHyp [a] (-1)

1: .....wf..... NILNIL

$\vdash a \in \mathbb{N}$

2:

7.  $\{m: \mathbb{N} \mid$   
 $\forall k: \mathbb{N}.$   
 $(a + b = \text{fib}(k))$   
 $\Rightarrow ((k \leq 0) \Rightarrow (a = 0))$   
 $\Rightarrow ((0 < k) \Rightarrow (a = \text{fib}(k - 1)))$   
 $\Rightarrow (m = \text{fib}((n - 1) + k))\}$
- $\vdash \{m: \mathbb{N} \mid$   
 $\forall k: \mathbb{N}.$   
 $(a = \text{fib}(k))$   
 $\Rightarrow ((k \leq 0) \Rightarrow (b = 0))$

$$\begin{aligned} &\Rightarrow ((0 < k) \Rightarrow (b = \text{fib}(k - 1))) \\ &\Rightarrow (m = \text{fib}(n+k)) \end{aligned}$$